Unsteady Inviscid and Viscous Computations for Vortex-Dominated Flows

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Abstract

THE unsteady supersonic flow around a rigid sharp-edged delta wing is solved using the unsteady Euler and thin-layer Navier-Stokes equations. The problem is formulated relative to a moving frame of reference so that computation of the grid motion is not required. A three-dimensional, implicit, aproximately factored, central-differencing, finite-volume scheme is used to obtain the time-accurate, unsteady, locally conical flow. The inviscid and viscous results are compared during the periodic response of a wing undergoing forced rolling-oscillation motion. The results show substantial differences in the surface pressure results and other distributed flow characteristics. These differences are attributed to the inviscid and viscous flow predictions in the vortex-dominated regions. The results of the total loads do not show any significant differences.

Contents

Governing Equations

The unsteady flow problem around a rigid wing is formulated relative to a moving frame of reference. The advantage of this formulation is that the body-conformed grid will be time independent. Using the transformation equations from the space-fixed to the moving frame of reference,[1] the conservative form of the dimensionless, unsteady, compressible, thin-layer, Navier-Stokes equations in the index notation is given by

$$\frac{\partial \bar{Q}}{\partial t} + \frac{\partial \bar{E}_m}{\partial \xi^m} - \frac{\partial (\bar{E}_v)_3}{\partial \xi^3} = \bar{S}, \qquad m = 1,2,3 \tag{1}$$

where

$$\bar{Q} = (1/J)[\rho, \rho u_1, \rho u_2, \rho u_3, \rho e]^t \tag{2}$$

$$\bar{E}_m = (1/J)[\rho U_m, \rho u_1 U_m + \partial \xi^m p, \rho u_2 U_m + \partial_2 \xi^m p, \rho u_3 U_m + \partial_3 \xi^m p, \rho h U_m]^t \tag{3}$$

$$(\bar{E})_3 = (1/J)[o, \partial_k \xi^3 \tau_{k1}, \partial_k \xi^3 \tau_{k2}, \partial_k \xi^3 \tau_{k3}, \partial_k \xi^3 (u_n \tau_{kn} - q_k)]^t, \qquad k = 1,2,3 \tag{4}$$

$$p = \rho(\gamma - 1)\left(e + \frac{V^2}{2} - \frac{V_t^2}{2}\right), \qquad h = \frac{\gamma p}{\rho(\gamma - 1)} + \frac{V^2}{2} - \frac{V_t^2}{2} \tag{5}$$

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In Eqs. (1-5), ξ^m is a time-independent body-conformed coordinate; $\bar{S}$ a source term due to rigid-body motion[1]; ρ the density; p the pressure; u_1, u_2, and u_3 the Cartesian components of the fluid relative velocity; $\bar{V}$ the fluid relative velocity; $\bar{V}_t$ the transformation velocity of the moving frame; e and h the total energy and enthalpy per unit mass relative to the moving frame; U_m the contravariant component of the fluid relative velocity; τ_{kn} the components of the shear stress tensor; J the Jacobian of metric coefficients; q_k the conduction heat-flux component; and γ the ratio of specific heats. In the Euler limit, the viscous and heat-conduction flux $(\bar{E}_v)_3$ is set equal to 0, and the unsteady Euler equations in the moving are obtained.

Boundary and Initial Conditions

For the freestream Mach number range considered, a conical shock is enclosing the wing. Therefore, the computational domain is extended to a size such that the shock is captured as a part of the solution. The flow conditions outside of the shock are those of the freestream conditions relative to the moving frame. At the solid surface, the pressure boundary condition is obtained from the normal momentum equation, and the temperature boundary condition corresponds to an adiabatic condition. At the plane of geometric symmetry, a periodic boundary condition is used. The initial conditions for the flow relative motion are obtained by subtracting the terms due to the impulsively started motion of the moving frame from the absolute steady motion.

Computational Applications

An implicit, approximately factored, central-differencing, finite-volume scheme[2] is used to obtain the locally conical flow solutions. The conical flow solution is obtained from the three-dimensional scheme by forcing the absolute conservative components of the flow vector field to be equal on the planes at $x = 0.95$ and 1.05. The scheme for the Euler and Navier-Stokes equations is applied to a rigid sharp-edged delta wing of a sweep-back angle β of 70 deg, at a mean angle of attack $\alpha_m = 10$ deg and a freestream Mach number $M_\infty = 2$. The freestream Reynolds number R_e for the viscous solution is 0.5×10^6. The body conformed grid is generated by using a modified Joukowski transformation of 264×90 cells (around and normal to the wing) for the Navier-Stokes equations and 132×60 cells for the Euler equations. It should be noted here that the solution for the Euler equations does not require the same level of grid fineness as that of the Navier-Stokes solution. The CRAY2 computer of the Numerical Aerodynamic Simulation (NAS) facilities at the NASA Ames Research Center was used to produce the present results. The wing was given a rolling sinusoidal oscillation of $\bar{\omega}$ and θ of

$$\bar{\omega} = -\omega_0 \cos kt\hat{i}, \qquad \theta = -\theta_{\max} \sin kt \tag{6}$$

where $\theta_{\max} = \omega_0/k$ is the maximum amplitude of roll angle, k the dimensionless reduced frequency ($k = k^* 1/U_\infty$, where k^* is the dimensional frequency and 1 is the root chord), and $\hat{i}_1$ a unit vector parallel to the x axis of the moving frame of reference. In the present case, $\theta_{\max} = \pi/12 = 15$ deg, $k = 1.337$, and

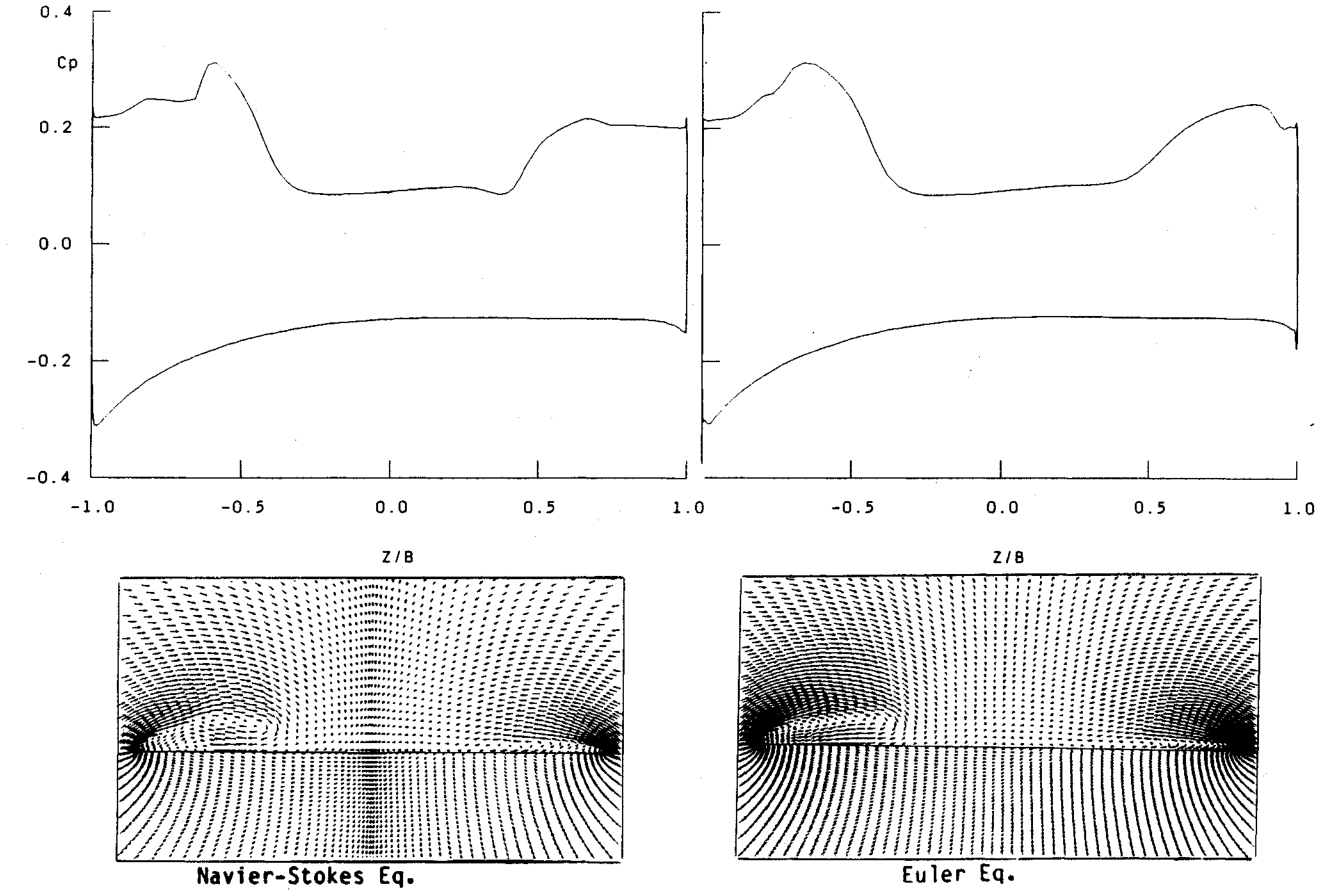

Fig. 1 Distributed flow characteristics at $t = 9.9875$, $\theta = -10.607$ deg↓; surface-pressure coefficient and cross-flow velocity.

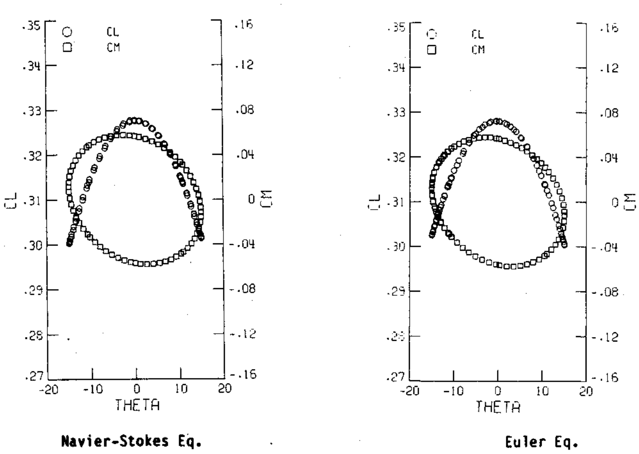

Fig. 2 Total lift and rolling-moment coefficients during the periodic response of flow.

$\omega = 0.35$. The corresponding period of oscillation is 4.7. For the Navier-Stokes solution, $\Delta t = 0.00075$, and hence each cycle of oscillation takes 6267 time steps. For the Euler solution, $\Delta t = 0.0025$, and hence each cycle of oscillation takes 1880 time steps.

Figure 1 shows a comparison of the results of the distributed characteristics at $t = 9.9875$, where the roll angle is -10.607 deg and the wing is rolling in the counterclockwise direction. On the upper surface, the suction pressure on the left leading edge shows a larger suction peak than that on the right leading edge. In the Navier-Stokes results on the left leading edge, one can see the primary and secondary vortices that are separated by a shock under the primary vortex. The corresponding Euler results show a longer primary vortex, no secondary vortex, and a weak shock under the primary vortex. Additional details of these results are given in Ref. 2.

Figure 2 shows the results of the lift and rolling-moment coefficients vs the roll angle during the periodic response of the third cycle. For all engineering purposes, the Navier-Stokes and Euler results are identical. The rolling-moment coefficient shows a typical hysteresis response, whereas the lift coefficient shows very slight hysteresis response.

Thus, it is concluded that for the accurate prediction of distributed aerodynamic characteristics, the Navier-Stokes equations are required, at least in the vortical-shock interaction region. But for the prediction of the total aerodynamic loads, the Euler-equations solution is sufficient, providing that the leading edge is sharp.

Acknowledgment

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References

[1]Kandil, O. A., and Chuang, H. A., "Unsteady Navier-Stokes Computations Past Oscillating Delta Wing at High Indcidence," AIAA Paper 89-0081, Jan. 1989.

[2]Kandil, O. A., and Chuang, H. A., "Comparison of Unsteady Euler and Navier-Stokes Computational Results for Vortex-Dominated Flows," *Proceedings of the Royal Aeronautical Society,* London, UK, April 1989, pp. 25.1–25.16.